Large Transverse Momenta and Tsallis Thermodynamics

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Abstract. The charged particle transverse momentum (p_T) spectra measured by the ATLAS and CMS collaborations in proton - proton collisions at $\sqrt{s} = 0.9$ and 7 TeV have been studied using Tsallis thermodynamics. A thermodynamically consistent form of the Tsallis distribution is used for fitting the transverse momentum spectra at mid-rapidity. It is found that the fits based on the proposed distribution provide an excellent description over 14 orders of magnitude with p_T values up to 200 GeV/c.

1. Introduction

It is by now well-known that the Tsallis distribution gives excellent fits to the transverse momentum distributions observed at the Relativistic Heavy Ion Collider (RHIC) [1, 2, 3] and at the Large Hadron Collider (LHC) [4, 5, 6, 7, 8] with only three parameters, q, T and dN/dy (or, alternatively, a volume V [9, 10, 11]). The parameter q is referred to as the Tsallis parameter and discussed elsewhere [12] in detail.

It was recently shown that these fits extend to values [14] of p_T up to 200 GeV/c [15, 16]. This is unexpected because in this kinematic range hard scattering processes become important [17]. A description of the high p_T results has been discussed in [18] where a model using a combination of Tsallis at low p_T and QCD hard scattering at high p_T was considered. The present analysis shows that the Tsallis distribution describes measurements up to the highest p_T using the same parameters as those obtained at low p_T.

A power law based on the Tsallis distribution [13] is used to fit the p_T spectra of charged particles measured by the ATLAS and CMS collaborations. The ATLAS collaboration has reported the transverse momentum in an inclusive phase space region taking into account at least two charged particles in the kinematic range $|\eta| < 2.5$ and $p_T > 100$ MeV [7]. The CMS collaboration has presented the differential transverse momentum distribution covering a p_T range up to 200 GeV/c, the largest range ever measured in a colliding beam experiment [14].

The results can be compared to those obtained in [15, 16, 17, 19, 20] where very good fits to transverse momentum distributions were presented. We confirm the quality of the fits but obtain different values of the parameters albeit using a different version of the Tsallis model.

2. Tsallis Distribution

The Tsallis distribution is defined as

$$f(E) \equiv \left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}}, \tag{1}$$

which at large energies behaves as

$$\lim_{E\to\infty} f(E) = \left(\frac{E}{T}\right)^{-\frac{1}{q-1}}, \tag{2}$$

so that the scale is set by T and the asymptotic behaviour is set by q. For high energy physics a consistent form of Tsallis thermodynamics for the particle number density n, energy density, ϵ, and pressure P are given by [22, 23]

$$\begin{aligned} n &= g\int\frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, &(3)\\ \epsilon &= g\int\frac{d^3p}{(2\pi)^3}E\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, &(4)\\ P &= g\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. &(5)\end{aligned}$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. This introduces only one new parameter q which for transverse momentum spectra is always close to 1. The consistency conditions

$$d\epsilon = Tds + \mu dn, \qquad\qquad dP = nd\mu + sdT, \tag{6}$$

are satisfied. This is shown explicitly for one of the relation $n = \partial P/\partial\mu$.

$$\begin{aligned} \frac{\partial P}{\partial\mu} &= g\int\frac{d^3p}{(2\pi^3)}\frac{p^2}{3E}\frac{\partial}{\partial\mu}f^q\\ &= -g\int\frac{d^3p}{(2\pi^3)}\frac{p^2}{3E}\frac{d}{dE}f^q\\ &= -g\frac{4\pi}{(2\pi^3)}\int_0^\infty dp\,\frac{p^4}{3E}\frac{d}{dE}f^q\\ &= -g\frac{4\pi}{(2\pi^3)}\int_0^\infty dp\,\frac{p^3}{3}\frac{d}{dp}f^q \qquad \text{using} \qquad EdE = pdp\\ &= g\frac{4\pi}{(2\pi^3)}\int_0^\infty dp\,p^2f^q\\ &= n\end{aligned}$$

3. Transverse Momentum Distributions

From the expression for the total number of particles:

$$N = gV\int\frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{7}$$

we obtain the corresponding momentum distribution

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \tag{8}$$

In terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, this becomes (at mid-rapidity $y = 0$ and for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\ dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}, \tag{9}$$

and, for charged particles it is given by a sum over the most abundant ones, $\pi^\pm, K^\pm, p, \bar{p}$. The results are shown in figures (1) and (2) and have been discussed in more detail in [24].

$$\left.\frac{d^2N(\text{charged})}{dp_T\ dy}\right|_{y=0} = \sum_{i=\pi,K,p,\ldots} g_i V\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}, \tag{10}$$

The resulting parameters are listed in table 1.

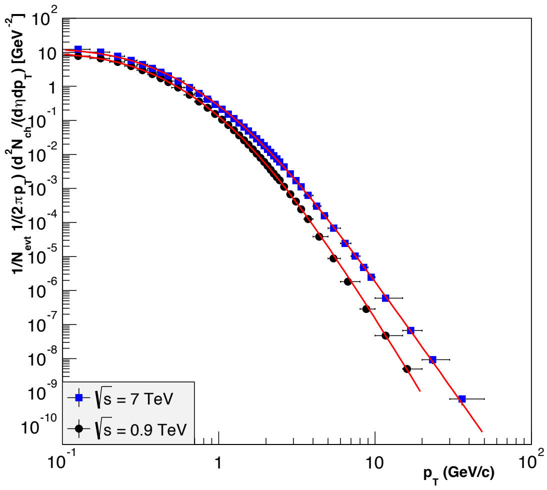

Figure 1. Charged particle multiplicities as a function of the transverse momentum measured by the ATLAS detector for events with $n_{ch} \geq 2$, p_T >100 MeV and $|\eta| < 2.5$ at $\sqrt{s} = 0.9$ and 7 TeV in proton - proton collisions [7] fitted with Tsallis distribution.

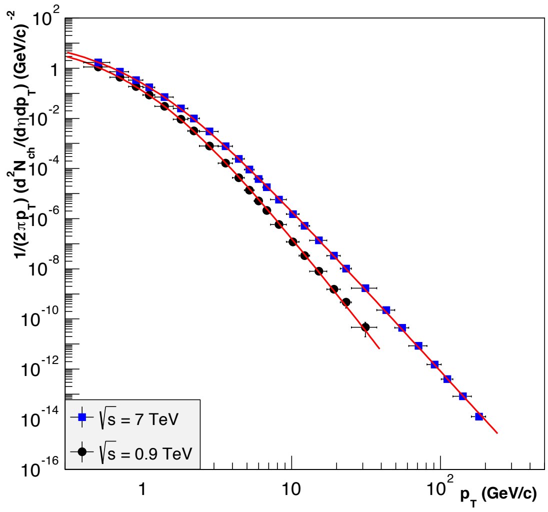

Figure 2. Charged particle differential transverse momentum yields measured within $|\eta| < 2.4$ by the CMS detector in proton - proton collisions at $\sqrt{s} = 0.9$ and 7 TeV [14] fitted with Tsallis distribution.

4. Conclusion

It is quite remarkable that the transverse momentum distributions measured up to 200 GeV/c in p_T can be described consistently over 14 orders of magnitude by a straightforward Tsallis distribution. The advantages are that the thermodynamic conistency conditions are satisfied:

$$n = \frac{\partial P}{\partial \mu} \qquad \text{etc...},$$

and the parameter T truly deserves its name since $T = \partial E/\partial S$.

Table 1. Values of the q, T and R parameters and χ^2/NDF obtained from fits to the p_T spectra measured by the ATLAS [7] and CMS [14] detectors.

Experiment	$\sqrt{s}$ **(TeV)**	q	T **(MeV)**	R **(fm)**	χ^2/NDF
ATLAS	0.9	1.129 ± 0.005	74.21 ± 3.55	4.62 ± 0.29	0.657503/36
ATLAS	7	1.150 ± 0.002	75.00 ± 3.21	5.05 ± 0.07	4.35145/41
CMS	0.9	1.129 ± 0.003	76.00 ± 0.17	4.32 ± 0.29	0.648806/17
CMS	7	1.153 ± 0.002	73.00 ± 1.42	5.04 ± 0.27	0.521746/24

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